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Risk—A Fundamental Condition of Doing Mathematics

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Abstract: The theme of this special issue is risk. But risk is not a common topic of investigation in mathematics education, lest it be an occasional interest in “at risk” students, generally defined as those who likely will fail at school. In this study, we are not interested in this rather limited use of the risk concept. Instead, we show that risk not only is a condition of human life generally, but also a necessity for teaching and learning mathematics. To show this, we develop the concept of risk with materials from a second-grade mathematics unit on geometry. Implications are drawn concerning the particular forms of ethics that take into account the risk incurred by teachers and students when doing mathematics in the classroom.

Keywords: curriculum and lesson planning, learning outcomes, instruction, learning, teaching, epistemology, ethics.

To Live is to Risk

[T]o act or even to live already is to accept the risk of infamy with the change at glory. (Merleau-Ponty, 1960, p. 116)

The act includes a sacrifice and a risk. . . . This is the risk: the primary word [*Thou*] can only be spoken with the whole being. (Buber, 1970, p. 10)

To be together, or to be in common, therefore, is the proper mode of being of existence as such, which is the mode where being as such is put into play, where being as such *is risked or exposed*. (Nancy, 1993, p. 155)

To live means to risk. To live means to *experience*; and to *experience*, as the etymology of the syllable *per* indicates, means to face *peril* (Roth, 2013).¹ To act, to speak, and thereby to experience inherently means to risk because, when individuality “commits itself to the objective condition in putting itself into a deed, [it] does of course risk being altered and perverted” (Hegel, 1806, p. 258). This is especially so because what becomes of acting and speaking, inherently, is unknown and, therefore, inherently is associated with peril. Whereas it is true that “the individual, who has not dared his life, can be recognized as *person*,” this person “has not attained the truth of this recognition as an independent self-consciousness” (p. 119). More recently, the very notion of agency also has been characterized in its essential character of risk: “life . . . [is] answerable, risk-fraught, and open becoming through performed actions” (Bakhtin, 1993, p. 9). Only technical rationality, the quintessence of metaphysics, cannot admit the equivalence of life and risk: both technological rationality and metaphysics presuppose the identity

¹ This aspect of human experience has found its entry into commonsense sayings as “Wer nicht wagt der nicht gewinnt [Who does not risk does not gain]” (ger.), “No guts, no glory” or “Who dares wins,” “La chance sourit aux audacieux [Chance smiles at the audacious]” (fr), “Chi non rischia, non rosica [Who does not risk does not nibble]” (it), or “Quem não arrisca, não petisca [Who does not risk does not nibble]” (pt).

of knowledge and Being (*Sein*) and therefore the predictability of life (Heidegger, 2006). In mathematics education, a vast majority of curricula, textbooks, or lesson plans still are employed as technologies, as means-to-ends (Maheux, 2011): metaphysical in their assumed transcending and ordering of the messiness of life.

Metaphysics is the human entanglement in the technical and in the will (Heidegger, 2009). In metaphysics—technique and cause–effect (will) reasoning—there exists a tendency or desire to eliminate risk, and perhaps the hope that risk can in fact be eliminated. This tendency, desire, and hope is embodied by technical thinking with its striving toward “functionalization, perfection, automatization, bureaucratization, information” (p. 60). The desire to technologize life is to submit it to control: “Technopoly is a state of culture. It is also a state of mind. It consists in the deification of technology” (Postman, 1992, p. 71) and, therefore, is the epitome of what has been called *the will to power* (Nietzsche, 1954a, 1954b, 1954c). Heidegger refers to it as *onto-theo-logical*, because the same desires underlie the beliefs in horoscope and pre-determination of life on the part of some deity, and the political aspirations of totalitarian states. But, as theorists increasingly recognize, technology and the modernization representing will to power are in fact sources of risk, including sources of risk to the self (Beck, 1992; Giddens, 1991).

Less ominous, the same kind of thinking exists in the practice of curriculum design, where the *Whats* and the *Hows* of learning are specified in advance of the mathematics curriculum as an open event of yet to be determined nature (Roth, 2013a), as if it were possible to plan mathematical learning administratively (Neyland, 2001). Because schooling as institution has the (societal) task to plan high school and college completion and the production of the qualifications for subsequent job opportunities, “learning has to be seen as something that *can be directly administratively planned*” (Holzkamp, 2013, p. 118). That is, in the institutional practice of curriculum specifications and the specific lesson plans—which determine in advance what students should know after a year in the mathematics class or after doing a task—we find the same kind of technical rationality that characterizes other parts of life in society. “Learning technologies” constitute but another means in the ill-fated attempt “[t]o make learning more efficient and more interesting” (Postman, 1992, p. 171).

On the other hand, we all have to recognize that doing mathematics is an expression of life. It is something we, human beings, do as part of our being-alive. In that sense, to live mathematics is to take risks, and to risk means to expose oneself to hazard or danger. When teachers and students engage in doing mathematics (together), they thus inherently expose themselves to all sorts of risks:

¹risk, noun

1: possibility of loss or injury: peril

2: someone or something that creates or suggests a hazard

3a : the chance of loss or the periods to the subject matter of a contract

b : a person or thing that is a specified hazard to an insurer

c : an insurance hazard from a specified cause or source

4: the chance that an investment . . . will lose value

– at risk

: in a state or condition marked by a high level of risk or susceptibility

²risk, verb

1: to expose to hazard or danger

2: to incur the risk or danger of

(from Merriam Webster Online dictionary, 2014)

Translations

Risiko (ger., neut), risque (fr., m), rizq (arab.), ρίσκο (risko, gr., neut), риск (risk, rus., m), rischio (Ital., m), ryzyko (pol., neut), risco (por., m), riziko (slv., m), riesgo (spa., m), risk, riziko (tur.)

The theme of this special issue is risk. Saying to risk, as the dictionary definitions show, is saying to expose oneself to hazard or danger, to engage the possibility of peril. Someone or something that constitutes a hazard is *a risk*. Interestingly, as the translations show, the term *risk* is one word that with little change exists in many, very different European languages. Yet risk is not a common topic of investigation in mathematics education, other than the occasional interest in *at-risk* students (e.g., Karsenti, 2010). *At risk*, according to the dictionary definitions quoted above, is used in (mathematics) education as being “in a state or condition marked by a high level of risk or susceptibility.” In the context of education, *at-risk students* tend to be defined as those who likely will fail at school. In this study, we are not interested in this rather limited use of the risk concept. Instead, we argue that risk ought to be recognized as a condition of human life generally and of mathematics education specifically. From this specification arise particular forms of ethics that take into account risk as a condition of the living mathematics curriculum.

Risks, Plans, and the Living Curriculum

Many characteristics ascribed to humans do in fact exist among other animals: culture, tool use, division of labor, exchange (food for sex), and distribution (Roth, 2009a). However, whereas, for example, a bee humiliates many human builders with its precision in constructing a honeycomb, human beings uniquely demonstrate a capacity to plan ahead.² That is, a distinguishing feature of human beings has to do with how we appear to control our life conditions through forward thinking or, as Marx and Engels (1962) put it: “the worst builder excels the best bee in having built the cell in his head in advance of building it from wax” (p. 193). Curriculum planning is part of a technology of control over classroom events, and teachers and researchers are familiar with the ways in which mathematics curricula are specified by the relevant educational authority. Thus, for example, in the Canadian province of British Columbia, there are instructional resource packages specifying what students are expected to do (such as describing, comparing, and constructing two-dimensional shapes including rectangles and squares) along with prescribed learning outcomes and suggested achievement indicators (Figure 1). Inasmuch, curriculum planning is also controlling what counts as legitimized knowledge to be acquired (Postman, 1992), indicating what (all) students are expected to achieve (in this case: the identification of common attributes of typical two-dimensional shapes, such as rectangles, from sets of the same type and create a pictorial representation of a given shape).

Inherent in such curriculum specification—and in the associated lesson plans of teachers—is the assumption that learning can be preplanned. As a consequence, the real subject of the process begins to change: “the subject of learning would not be the pupil, but the teacher, who has to plan and perform the lessons in such a way that her/his teaching efforts are directly measurable and controllable by the effects they produce in the pupils” (Holzkamp, 2013, p. 118). Even more so when teachers are expected to treat all students the same, exposing everyone to the same curriculum, outcome differences can be and are explained by means of students’ different abilities. This way of thinking about the curriculum does not account, however, for the abyss that exists between *any* plan and the situated actions that it is designed

² There is some evidence, though, that chimpanzees have such capacity, as seen in the fact that they may carry tools for 50 km from one location to another to crack nuts (Roth, 2009a).

to control (Suchman, 2007). Even the most experienced scientists think they act or have acted according to a plan only to find out later that what they have done was not what they thought to have done (Roth, 2009b). That is, there is an *eo ipso* difference between any lived curriculum and its prefixed form (Pinar & Reynolds, 1992), which evidently implies the persistent present of risk that the intended form is not realized in the living curriculum (Roth, 2013c).

SHAPE AND SPACE	
3-D Objects and 2-D Shapes	
Prescribed learning Outcomes	Suggested Achievement Indicators
<p><i>It is expected that students will:</i></p> <p>C8 describe, compare, and construct 2-D shapes, including</p> <ul style="list-style-type: none"> – triangles – squares – rectangles – circles [C, CN, R, V] <p>[C] Communication [CN] Connections [R] Reasoning [V] Visualization</p>	<p><i>The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.</i></p> <p><i>Students who have fully met the prescribed learning outcome are able to:</i></p> <ul style="list-style-type: none"> <input type="checkbox"/> sort a given set of 2-D shapes and explain the sorting rule <input type="checkbox"/> identify common attributes of triangles, squares, rectangles, and circles from given sets of the same type of 2-D shapes <input type="checkbox"/> identify given 2-D shapes with different dimensions <input type="checkbox"/> identify given 2-D shapes with different orientations <input type="checkbox"/> create a model to represent a given 2-D shape <input type="checkbox"/> create a pictorial representation of a given 2-D shape

Figure 1. Constructed based on the student achievement specifications matching *prescribed learning outcomes* with achievement indicators from the mathematics curriculum for second grade students in the province of British Columbia. (from MoE-BC, 2007, pp. 45 & 110)

At yet another level, it is accepted within different theoretical approaches that students “learn” based on what they already “know” not in the least because their “interpretations” and “constructions” are a function of their previous experiences. In more general terms, acting (including reflecting) expands a person’s action possibilities; much of this expansion is not and cannot be anticipated precisely because what will have been learned *inherently* cannot have been the intended object, because it is unknown until revealed in the course of learning, whether the subject is a second-grade mathematics student (Roth, 2012) or one of the great “poets,” including Yeats, Galileo, and Hegel (Rorty, 1989). But if what we come to know is unforeseeable, then there is risk. This is so not only for the student—who cannot predict the learning outcomes are, whether s/he is on the right track to get there, and whether what s/he is doing is in fact part of the required trajectory—but also for the teacher, who is never assured that any specific lesson is going to lead some, many, most, or all students to exhibit actions that are consistent with the expected achievement indicators. But as the living curriculum unfolds, teachers and students

present one another with achievements and expectations, which provide them with opportunities to navigate the inherent gap between the plans for and outcomes of the curriculum. As a result of their transactions, teachers learn and students teach to the same extent that teachers teach and students learn: all participants expand their room to maneuver (Roth & Radford, 2010). But the risk of failing still exists, even in the most well-thought and planned curriculum. This so is to a great extent because the living curriculum constitutes an unfinished event*-in-the-making whose specific nature can be determined and grasped until *after* it has ended and therefore no longer exists as event*-in-the-making (Roth, 2013a).

A recently published book-length study of mathematical learning from a cultural-historical activity theoretic perspective provides descriptions of student engagement that clearly exhibit different responses to the risk in mathematics classrooms (Roth & Radford, 2011). On the one hand, we see Aurélie engaging with the task for a while, and then states that what she is doing makes no sense, before, a few moments later, she concludes: “I don’t understand and I will never understand.” Aurélie pounds on the table and throws herself against the backrest. For the remainder of the lesson she slouches in her chair or has her head buried in her arms on the table. In any event, it appears as if she no longer engaged other than to copy what her group mate Thérèse had done. Mario, on the other hand, continues despite what looks like extreme frustration, and despite articulating failure to understand what he—unbeknownst to him at the time—is yet to come to understand. He also engages in a relation with the teacher; and, while so doing, both exhibit frustration: Mario for not knowing, the teacher apparently for not finding a way to help. In the end, however, Mario breaks through the impasse and eventually states, “Me, I now understand” (p. 89). At this point, the risk of failing has passed, the task is completed, a goal (that of understanding) is achieved: there is mastery possible only over the old and never over the radically new (Romano, 1998).

This description shows that to get to a point of understanding, a student *has to* engage with the task without knowing whether s/he will get to such a state. There is no guarantee beforehand that Mario will get to the point—in this lesson or in any lesson that follows—where he can actually state to be understanding. That is, there is an inherent risk that all the engagement, all the (intellectual, affective) investment made will get him anywhere, especially near the intended (planned) state identified in the curriculum. Not engaging, while avoiding the risk of failing in the task, actually means never reaching its intended goal. The risk of overall failure is even greater: the risk of mutism (Romano, 1998). Here, the risk is that of an inability to speak in mathematics and mathematically. In Aurélie’s statements, this risk is co-articulated, for she says that she will never understand, which, in the context, can be heard as saying that she will never understand whatever investment she makes (definition 4 above). But by not engaging, never understanding is a certainty—unless there were to be some form of immaculate and miraculous conception. From a certain perspective, the risk of failing is low; but, unacknowledged, the risk of failure is even higher at the same time. Thus, whereas speech constitutes “a conquest over silence, which nevertheless echoes in it as its most intimate risk omnipresent in all Saying” (p. 226), the making of sense has non-sense as its necessary companion.

Risks in the School Mathematics Curriculum: A Contribution to Theory Development

This study is designed as a contribution to building an epistemology that takes into account *risk* as an irremediable character of life generally and as an irremediable character of engagement in the mathematics classroom specifically. In the pursuit of this endeavor, we provide in this section exemplifying descriptions and analyses from a geometry unit for second-grade students. The lessons were to be consistent with the official, British Columbia curriculum, here with Prescribed Learning

Outcome C8 and the associated achievement indicators (Figure 1). In this section, we first provide a description of the lesson background and then present the micro-analytic dimensions of risk arising from speaking and responding; this is followed by the articulation of a meso-analytic perspective on risk at the lesson level.

The Lesson Background

The specific lesson tasks were design by a university-based mathematics educator (pseudonym Mrs. Tran) with extensive experience as a teacher at the second-grade level and the regular classroom teacher, who also served as vice-principal of the school and as a participant on governmental curriculum committees. The second-grade students represented, from the school perspective, a broad range of cultural backgrounds and ability levels including some “learning disabled” students. (Extensive background is provided in Roth, 2011a.) For the particular segment of the curriculum unit, the teachers had designed a task in which the children first used a loop of string of a specified length for the purpose of identifying all possible rectangles that could be contained within the loop given that the string had to be run along the lines of a square grid paper. Students were then to make a trace along the string to generate a pictorial representation of the shape. In this way, the students were involved in a task that would offer them opportunities to exhibit at least two of the achievement indicators that go with the prescribed learning outcome C8 (Figure 1). The two teachers showed students how they should work together to tack the string of a chosen configuration to their paper so that it would be easier to subsequently trace it (Figure 2).

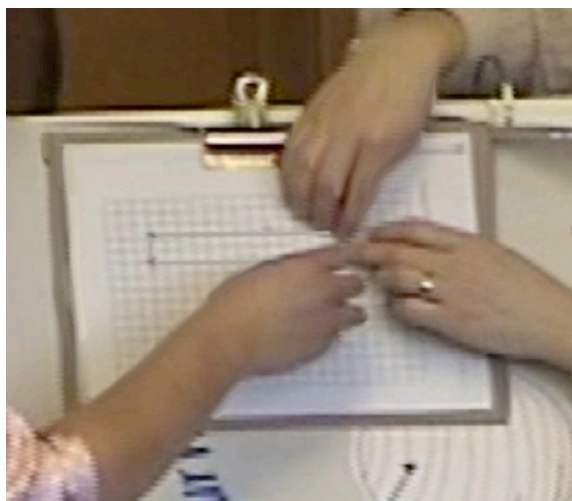


Figure 2. The two teachers—Mrs. Tran to the left, the regular classroom teacher on the right—are showing students how to lay out a string to make rectangles and fix these on a coordinate system so that they can count how many squares they contain and measure their lengths and widths

During the subsequent lesson, Mrs. Tran had different students come to the board to identify a rectangle among those available on the tray of the board that had been among the ones they had found. In the end, there were seven rectangles on the board, each containing a number that specified the number of unit squares in the rectangle (Figure 3). Using a piece of chalk, Mrs. Tran had added the lengths and widths of each rectangle in terms of the number of squares making the side. She asked students to discuss in their groups what was the same about all of the rectangles on the chalkboard or on their working sheet. After a while she brought the students back together into a whole-class setting. Even

before the class had settled, she (TT in the transcription) was calling on Thomas, who apparently had raised his hand.³

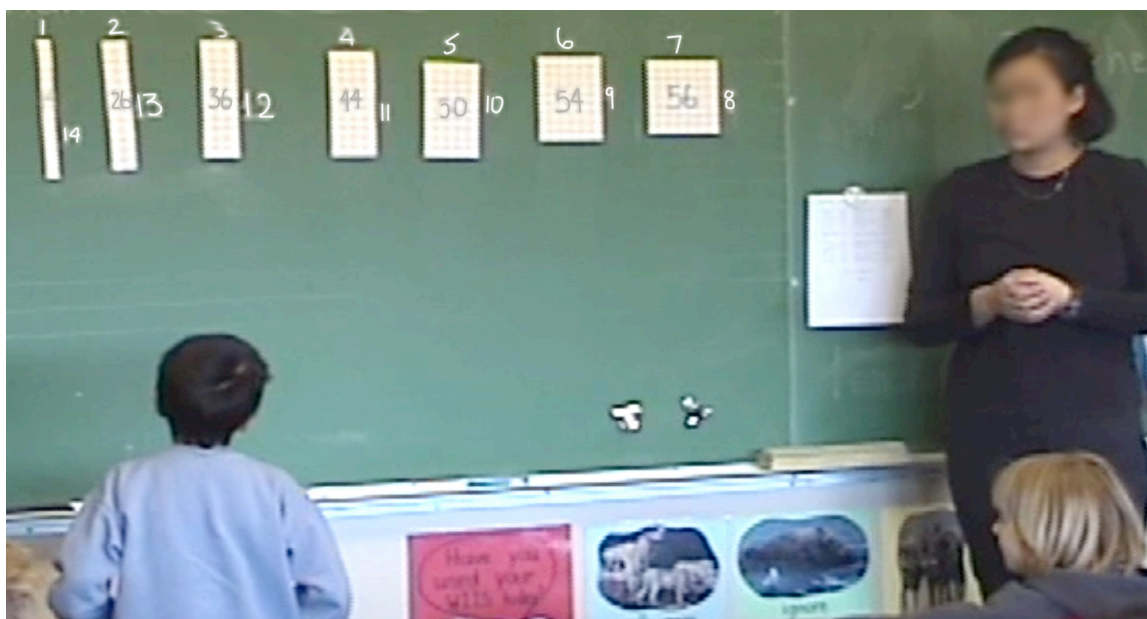


Figure 3. Mrs. Tran and Thomas at the chalkboard where there are seven rectangle displayed all produced by a string with fixed length

Risk in Speaking

01 TT: ((time on tape: 0:20:20)) you=re listening to thomas right now ((points towards Thomas))
sh:: ((places her right index finger against her mouth)) thomas <<all, h>what is the> ↓
same about A::LL of [these ↓ RECTangles.]

[[((pointing gesture from left to right along three of the rectangles, Figure 3)]]

The common approach to the analysis of verbal transcription makes two assumptions that close analyses of speaking have not borne out: (a) the thought is preformed and merely exits via the mouth as a sort of translation mechanism and (b) the social action of a statement is given with the statement (e.g., a question is asked). Thus, a traditional analysis of turn 01 from the lesson fragment might state that Mrs. Tran is calling on the students in the class to attend to Thomas, whose turn it is right then, and that she then asks Thomas a question about an invariant property of the shapes on the board. Such an analysis, however, does not take into account the risk difference between plan and situated action: Whether we have done or said what we—however vaguely or explicitly—intended to do or say is an empirical matter (e.g., Suchman, 2007). Speaking is risking, for

without a primary exposure to the very event of speech setting itself forth, without that opening to the new and ungraspable possibilities that grasp us in each speech by

³ The following transcription conventions are used: all words appear in small letters; RECTangles: capitalization for emphasized parts; ((points)): ethnographic observations; (2.20): pause in seconds; [,]: square brackets to indicate overlap; << > >: triangular brackets to indicate voice quality, all = allegro (fast), h = high pitch, len = lento (slow); ↓: step down in pitch; sh:: colon to indicate lengthening of sound; (.) unmeasured pause; →: reference to image on the right; =: equal sign for latching sounds; punctuation at the end of speech units indicate pitch movement, “.” and “,” for strongly and slightly falling pitch contour, “,” and “?” for slightly and strongly rising pitch contour.

disarming our powers over it, without that *risk, consubstantial with any genuine speech*, of sinking into silence and inability to speak, it would be impossible . . . to write a single sentence. (Romano, 1998, p. 225, emphasis added)

Traditional analysis of mathematics classroom talk thereby confuses the *Saying* with what will have been issued from it, the *Said*. While the *Saying* unfolds, nobody, not even Mrs. Tran, can know what she will have said once her *Saying* will have come to an end. This is so because “if speech presupposed thought . . . we could not understand . . . why the thinking subject himself is in a kind of ignorance of his thoughts as long as he has not formulated them for himself” (Merleau-Ponty, 1945, p. 206). At the level of the utterance, this is a moment at which risk no longer exists for the speaker: the leap is taken, the deed is done.⁴ Thus, rather than investigating what Mrs. Tran will have said, that is, the finished statement in turn 01, we need to analyze the *Saying* as it unfolds and *before* what ultimately will have been said is available (Roth, 2013a).

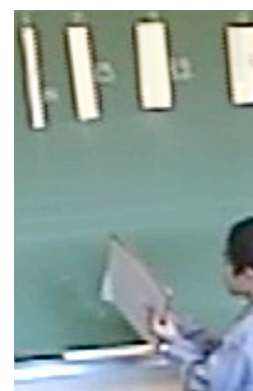
The need to analyze the *Saying* as it unfolds rather than the *Said*, which is available only when the speaker has ended and someone else speaks (acts) is quite evident from what follows because only 0.39 seconds later, Mrs. Tran continues to speak: “they all have” (turn 03). Whereas the statement in turn 01 is grammatically complete, the statement in turn 03 is grammatically incomplete. In fact, there have been suggestions that mathematics teachers produce *designedly incomplete statements* with the underlying intent that students complete these (Roth & Thom, 2009). That is, we may say that two different possibilities are offered for the nature of the next *Saying*. In the first, we may hear a question even though the pitch decreases towards the end of the *Saying* as this is common in constative statements rather than in questions. In the second, an unfinished statement is provided, which invites to be completed especially if there were a pause.

02 (0.39)

03 they ALL ha:ve

04 [(0.46)]

[[((Thomas walks to left of the display; gaze moves from at about second and third, then moves towards his paper →))]



We therefore observe a change in the offering. If Mrs. Tran had an intention completely formulated in her mind and was simply externalizing it by means of her *Saying*, she would not have needed to make a second offering, or, alternatively what was the actual second offering would have been the only one. We see an apparent change *after* she has completed what comes to be the first offering; and it is in the completed *Saying* that the result of her *Saying*, the *Said*, has become available. Only then, when externalized from pure Being, are our thoughts available to us and can become an object of conscious awareness and reflection (Hegel, 1807). If Mrs. Tran had heard the pause in turn 02 as a non-response on the part of Thomas, this would then show that her intended action was inappropriate for addressing Thomas. But why would an experienced elementary teacher such as Mrs. Tran ask a question that is inappropriate for the student facing her? The answer is that Mrs. Tran, exemplifying thereby all teachers, cannot inherently know with absolute certainty whether a student does or can understand what

⁴ Risk, of course, still is present, as we see below, at the conversational level.

she will have said or what s/he will have heard before she speaks, and not even once her Saying has come to an end. This uncertainty with respect to what she is doing while speaking or acting is subject to a double risk: not only the risk of non-sense referred to above, but also the risk of a sense completely other than that intended (Merleau-Ponty, 1964), as we see in the next section.

Risk in Listening

A person can reply to another person's talk only if s/he is actively listening. But listening means opening up to receive. Listeners cannot know beforehand what is coming at them. But opening up to receive includes the risk of being hurt. Recipients, in listening (or reading), do get hurt—there does not appear to be a day without another report of bullying, where a person feels hurt, insulted, or embarrassed by words someone else has voiced. Any active listening, therefore, comes with risks. Any articulation, such as a question, comes with risks that cannot be anticipated: the recipient comes to be exposed, hurt, or insulted in addition to the positive surprises that a Saying will have brought about when it has come to a close and the Said is available.

Whether the offering is taken as such remains uncertain. We do know this from the everyday interactions with others where, after intending to say or ask something, the intended addressee replies saying something like “Why are you insulting me?” or “F... off!” Now the initial speaker has to deal with the fact that what had been said has been heard as an insult independent of what a speaker might have intended. The social situation is such that the (unintended but nevertheless felt) insult has to be dealt with. What comes is heard in response to the naming of something as an insult. If Mrs. Tran had been teaching in an urban school, the student reply to turn 01 might have been “Why don't you screw someone else?” That is, *because someone is listening*, actively having the words ring in his/her ears while these are produced in her mouth (Roth, 2014), Mrs. Tran cannot ever be certain about the effect that these will have, and she cannot ever be certain about what she will have said as long as her Saying continues. The idea that thought cannot know what it is until it has externalized itself is not new but has been articulated in dialectical philosophy (Hegel, 1807) in terms of a thought (in-itself) that can grasp itself only when external to itself (where thought is for itself). More recently, dialectical psychologists continued this line of theory by suggesting that “thought is not expressed in word, but it completes itself through it” (Vygotskij, 1934, p. 269). Thought is complete only in the word.

Following Thomas' orientation towards the left end of the row of rectangles, Mrs. Tran can be heard saying, “Oh, we are talking about all of them” while gesturing along the row. It is as if she were replying to a statement about one or a few of these rectangles. In fact, one transcriber had written “these two ones” in turn 04, as if hearing something to which Mrs. Tran replied in turn 05.

04 [(0.46)]

[[((Thomas walks to left of the display; gaze moves from at about second and third, then moves towards his paper →))]]

05 TT: oh were talking about ↓ A::LL of them
((motions to the cards with her right hand))
they A::LL have what.

06 (1.33)



For one, we see here how listening is not only on the side of the addressee, but also is an integral part of uttering, for the speaker not only listens to herself, but is in constant coordination with the

evolving nature of the environment that her active talk is producing. The payoff associated with immediate feedback does not, however, come without risk. Conversations can quickly run off topic, apparently stall, or become paradoxical as in the famous “Topaze effect” described by Guy Brousseau, in which a student’s dubious facial expression leads the teacher to answer himself the question he seems to be asking. In such a case, it is not only one “planned talk” (if such a thing ever exists) that can be compromised, but the whole teaching and learning project, especially when the teacher wishes to work from students’ observations instead of monopolizing the mathematical/conceptual communicational space (Maheux & Roth, 2014). Listening to another also comes with this sort of conceptual risk, because the other’s Saying is an offering that, by definition, *presents* something toward which the listener has to orient himself (should it only be by ignoring it) because it is a social fact however one stands with respect to it (Star, 1991). When Mrs. Tran says, “what is the same about all of these rectangles” and “we’re talking about all of them,” she opens up a space for Thomas to articulate an observation, but also offers a framing of what is to be talked about. Attending to what can be heard implies, for Thomas, the risk of having to answer to something which demotes or even contradicts his previous observations—or situates him as “not listening.” There could be a case in which he was, indeed, just about to comment on two of the figures, and is prevented doing it after Mrs. Tran’s emphasis on “all” the rectangles. In addition, researchers studying communication in mathematics increasingly insist on the artificiality of distinguishing between communicating with oneself and with another (e.g. Sfard, 2008). The risks of listening are fully present from the moment that something-to-be-listened-to is made available, including of course the non-verbal.

Up to now, Thomas has not yet talked, though he has walked closer to the left end of the array of rectangles (turn 04). But rather than having a turn at talk, it is Mrs. Tran who is speaking again. She makes a statement concerning the topic of the talk: “We are talking about all of [the rectangles].” She continues with a designedly incomplete statement, “They all have . . .” and inherently specifies that it is some thing or property that has to come next here marked by the earlier articulated interrogative pronoun *what*. Once Mrs. Tran has finished speaking, a pause unfolds. When it will have ended, it has been longer than the normal 0.8 seconds that teachers tend to provide on average for students to respond (Tobin, 1987), and it has been longer than the standard maximum silence in conversations of about 1.0 second (Jefferson, 1989). From the present perspective, the risks increase as the pause gets longer.

05 TT:oh were talking about ↓ A::LL of them ((motions along the rectangles with her right hand)) they A::LL have what.

06 (1.33)

07 T: they <<len>A:::LL:: HA:::v:::e;>

08 (8.30)

09 T: ((*shrugs shoulders, turns to Mrs. Tran*)) i dont know.

On the one hand, Mrs. Tran—as teachers in a study of classroom questioning revealed (Roth, 1998)—might risk putting a student on the spot, which for many gender-sensitive teachers becomes a particular issue when the turn involves a girl. Thus, for example, in that study a teacher had general guidelines for their questioning such as “Always wait 5 seconds before accepting an answer” (p. 370), but “felt uncomfortable waiting in the case of Renata and Carla, for she could ‘see their discomfort, sort of squirming’” (p. 370). That teacher did not wait as long as her rule for action suggested but turned to other students rather than letting a girl “squirm.” There is a risk also for the teacher, in that waiting for a student to respond fully conflicts with a rule such as *Keep a class going* (Roth, 1998). Mrs. Tran does indeed wait; and, after what will have been a short longish wait, Thomas begins to speak. In fact, we

might consider him to have already begun *responding* while he attended to Mrs. Tran's speaking, and the pause is part of the production of the reply (Figure 3). Thomas draws out the words "they all have" (turn 07), which will have lasted 2.18 seconds when he ended.

Risk of Responding

If listening can be conceptualized as an early part of the response, especially when attending is made visible to the speaker, it is also possible to think more specifically about the risk involved in the responding dimension of the act (including that of attending). In the end, 45 seconds will have been taken from the first call on Thomas to Mrs. Tran's request for him to sit down and to put his hand back up when he will have thought about something that all the rectangles have. That is, Mrs. Tran will have allowed that all of that time be spent on providing for Thomas to produce a reply to question intended to state an invariant property of the rectangles only to find out that Thomas does not know—and, perhaps, given that she said "think about something that they all have," he might not have done the required thinking at all.

When he stops talking, another speaking pause begins to unfold; and, when it ends, it will have been an extremely long pause of 8.30 seconds. In the words of the teachers in another research project, Mrs. Tran really has done a lot (everything?) to let Thomas respond fully at the risk of not keeping the class going. One might say that everyone else had to wait. But if Mrs. Tran had begun a speaking turn earlier, she also would have risked not giving Thomas the opportunity to respond fully and, therefore, not providing him with an opportunity of articulating his thinking and, in this, not affording him an opportunity of learning. But in waiting for such a long time, other risks increase: (a) the possibility that Thomas does not get to reply or reply in a manner inappropriate with respect to the curriculum and (b) the possibility that Thomas will say nothing at all or admit to not knowing. It is this latter that we will have been observed once Thomas has spoken again. "I don't know," he says (turn 09).

By raising his hand, Thomas has taken risks, including not being able to provide the reply that the teacher is intending and having to admit his ignorance. Thomas is in a situation that some individuals experience as having been put on the spot, to which he contributed through his hand raising, setting up the risk that he actually would be chosen. In fact, Mrs. Tran's statement invites him to sit down, an invitation that Thomas accepts in and through his actions of returning to his seat and sitting down. More importantly, perhaps, her statement "you think of something that they all have and put your hand back up" can be heard as saying that he had not done any thinking about the question that was to be answered: "What do they all have?" or "What is the same about all of these?" By asking for a turn, Thomas has taken the risk of failing. We have no access to the emotional reflection of the situation on the part of Thomas. We do know that Mrs. Tran tended to utter something in the form of a question, wait as student were raising their hands, and then called on one of those students who had raised their hands. In the present, she had invited Thomas and, once the class was called to order, Thomas had arrived in the front of the classroom and next to the chalkboard (Figure 1). Mrs. Tran articulated a statement, which, as we see from the actions of Thomas, will have been treated as a question.

Such analysis clearly shows how responding is an expression of the *responsibility* (response-ability) for what had been said, realizing it in a way or another, as a question or an insult and so on. Bakhtin (1986) insists that *any* utterance is a response, a link in the chain of communication created by all preceding utterances: "Each utterance refutes affirms, supplements, and relies upon the others, presupposes them to be known, and somehow takes them into account" (p. 91). Uttering or responding is thus taking the risk of contributing to continuation (or the momentary interruption) of a series of exchanges, a risk that cannot be avoided from the instant that a student or a teacher recognized him/herself as an addressee. When Mrs. Tran says "they all have what" and Thomas responds, "They all

have . . . I don't know," she makes available to him a verbal utterance to which he finds himself in the obligation to answer even if he does not know what to say, as his answer shows (the hearable hesitation and the concluding "I don't know"). We are not saying that that the teacher riskily puts her student on the hot seat, but that there is an inherent risk that comes with responding, and that any utterance has the potentiality to trigger a response if someone (including oneself) recognized him or herself as its (potential) addressee. In as much, Thomas's final words in the previous fragment, his "I don't know," constitutes, in turn, a statement that offers itself to Mrs. Tran (and other people in the room) to be responded to, a statement that places everybody in presence at risk.

In this particular instance, the risk of Thomas's response is perhaps even higher since the topic of the last few verbal exchanges involve "knowing" about 2D shapes at the end of a lesson where students could be expected to notice regularities, thereby demonstrating the success in what the teacher had planned. The whole lesson is somehow at stake through his admittance of not knowing, and Mrs. Tran's response to Thomas's affirmation can lead to very different outcome, whether she manages to overcome this impression of knowing or not-knowing. We all know instances where children leave a class saying things like "it was so confusing today . . . we didn't learn anything . . . why did we even do that?" We also know instances, where a teacher makes just the "perfect, magical" comment that allows all the pieces of a puzzle fall into place, and makes students feel like they really got to learn something that day.

Risk of Conversing

As we can see, teacher-student exchanges present inherent risks in a number of ways. First, what might be intended as a question may in fact expose a student, if he has to admit not knowing and, thereby, be exposed as not knowing (Maheux & Roth, 2014). There are reports that teachers are especially sensitive when their questions address girls (Roth, 1998). In a fourth-/fifth-grade science class, the two teachers directed four times more questions to boys than to girls in whole class discussions because they did not want to expose the latter. For example, one of the teachers told a boy to "'shut up' as part of her attempt to provide a safe environment for girls to engage in risky responses" (p. 368). Second, Mrs. Tran cannot not know whether asking Thomas to come to the front of the classroom is taking the lesson any further. What is it that their transactional exchange will have produced once they are done and Thomas is asked to return to his seat? What if the two were to continue and thereby produce a lengthy exchange only to be confronted—after everything has been said and done—with the fact that nothing articulated has moved the lesson any further in terms of the curriculum plan?

Conversing is realized the articulation of utterances whose outcome (including their meaning, and contribution to mathematics teaching and learning) are also produced in the flow of uttering/responding. As such, conversations also include the notion of *time*, where risk is situated both in the here and now of taking a leap "after all that happened," and in the projected open space of "all that can follow." In our episode, for example, the making and taking of time is apparent in the particulars of turn 07. That is, in the drawing out of the syllables, time is made, which makes it possible for other things to be prepared. In that same action, Thomas is taking (his) time to produce the two parts of the reply both in the pause and the production of the words. In this double belonging of the time to Mrs. Tran and Thomas, we are provided with an example of the idea of *social* action that cannot be reduced to individual action: any pause, as much as any single turn (see below), is to be theorized as a joint rather than an individual action. This leads us to a revised model of talk in which any part of a conversation has sociological and psychological, synchronic and diachronic dimensions (Figure 4). It is because each word belongs to a minimum of two people (Vygotskij, 1934) that signification itself is at stake, where sense always exists over against non-sense into which it may collapse. There are no

definitive a priori rules or laws that specify the sense of an unfolding Saying (Davidson, 1986); any sense *advenes* in the event of speech and therefore cannot be reduced to it (Romano, 1998).

The analysis of classroom talk focuses on speakers and tends to forget that at the very instant that a speaker speaks a listener listens. That is, every word exists twice, simultaneously: for the speaker and recipient. Our analysis has to take this into account (Roth, 2014). This simultaneous (synchronic) presence of the word is captured in a revised model of the exchange, where it constitutes the sociological dimension of speech (Figure 4). On the part of the recipient, *actively attending* to the other constitutes the first part of the response; the second part is comprised by the actual *reply* (Figure 4). In the unfolding of the response, there is actually a transformation where, from the respondent's perspective, the Saying of the other is transformed into a reply. This transformation constitutes the psychological dimension (Vygotskij, 2005); this dimension also is diachronic. It is not so much that the response occurs in time, but that responding is at the very origin of time: it is making and taking time. This aspect of the exchange is particularly visible in the speaking pause, which makes space for either participant to take the next speaking turn, and for the respective other an opportunity to take the listener's turn. Who knows whether this time is not going to be wasted? Who knows whether the current part of an exchange will not have been putting one or the other participant in a situation where s/he lost or risked losing face?

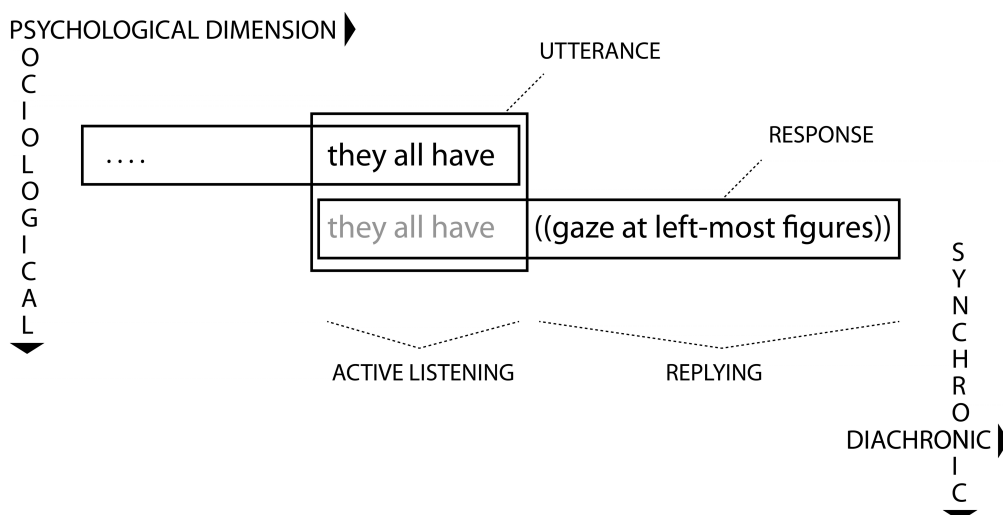


Figure 4. Every utterance is utterance only because it is both said and received; every response consists of active listening and replying.

Many philosophers agree that each act, including its negation the non-act, constitutes a risk (e.g., Bakhtin, 1993). In the preceding subsections, we exemplify risk as an essential dimension of each individual mathematics-oriented act and conceptualize the act in such a way it they cannot be reduced to the sum total of independent, individual locutions. Each action is the result of a dialogical interplay of turns in conversation, part of a dialogue (literally: “two-voices”) demanding that in each turn, both parties are involved: acting (speaking, raising hands) and attending (active listening, watching) crossing over into one another. Such acts are integral and constitutive parts of classroom episodes that have distinguishable beginnings and endings. For example, a student's turn is one such episode (Roth & Thom, 2009). In the lesson fragment drawn on here, it is Thomas's turn, which consists of a sequence of turns at talk. The outcome of the enchainment of *joint actions* that constitute the overall turn multiplies the risks. It is precisely the dialogical nature of the relation, here between teacher and student, that makes the outcome uncertain so that even novelist cannot know where the narratives are taking them (Bakhtin, 1984a). Thus, as we write above, when Mrs. Tran calls upon Thomas she cannot know

whether what will have been said has taken the lesson further towards achieving what the lesson plan has laid out.

Once and when Mrs. Tran is heard to be asking a question about what *all* the rectangles have in common, that is, once the content and the purpose of the statement becomes definitive, students' past and present perceptions are addressed. What is it that all of the rectangles on the chalkboard have in common? It is based on what students see—their perceptions—that students are expected to and will respond. This introduces a further dimension of our phenomenon because perception itself means risk: for it “would not know to win on one side without losing on the other and without exposing itself to the risk of time” (Merleau-Ponty, 1945, p. 396). Perception, then, rather than taking in the real and objective, means gaining and losing simultaneously, being exposed to the risk coextensive with life.

In the above-mentioned study in a fourth-/fifth-grade science course, even teachers with considerable experience at the grade level and substantive content related knowledgeability do not inherently get the timing of questions right or know when to stop a line of questioning (Roth, 1998). Teachers who are familiar with the students in their care have a better “sense for drawing the line between productive questioning and discouraging a child because their previous efforts [at soliciting an answer] appeared insufficient” (p. 364). Despite her considerable experience as a classroom teacher and despite her substantive knowledge of mathematics and mathematical learning as a professor of mathematics education, Mrs. Tran's questioning risks being inappropriate for the particular child in front of her (here Thomas). This inappropriateness may lead to exposing the child and to negatively affect his/her current emotional state specifically and the overall development in general.

Risk in the Mathematics Lesson

The dictionary definition of *risk* provided above includes the someone or the something that creates or constitutes a hazard. From the perspective of this definition, the British Columbia curriculum may be considered a risk because it is a setup for failure. Students may fail to exhibit the prescribed learning outcomes and teachers may fail to exhibit what teacher educators and researchers of teaching call *pedagogical*, *subject matter pedagogical*, and *general pedagogical knowledge*.⁵ In the preceding subsection, we present aspects of risk at the (micro-) level of individual and collective action, which really is the risk arising from living and relating as such. In this subsection, we focus on the teacher and the risks incurred in the very attempt to realize a previously articulated lesson plan. This is a meso-level analysis when we consider the particular goal of Prescribed Learning Outcome C8 in the context of the second-grade mathematics curriculum as a whole, and the effects of school mathematics on students in its entirety (e.g., loss of interest, completion of applied vs. academic level mathematics courses, opting out of mathematics-related careers). Here, at the meso-level, the curriculum was to get students to identify that the rectangles on the board all have the *same* perimeter, while *differing* in terms of the shape—long and skinny, almost square—and in terms of the number of unit squares they contain (i.e., area). Each time Mrs. Tran calls on a student to make a contribution generally and to respond to an apparent query—*What is the same about these squares?*—specifically, she incurs the risk of failing to receive the desired reply to the specific question at hand and to move closer to the intended end of the lesson at which point students should be doing what the curriculum specifies: describing similarities; describing and comparing (a set of) rectangles (Figure 1).

Following Thomas, Mrs. Tran calls on Lucas, again running the risk that what she is asking for, according to her lesson plan, is not going to be provided in and by the reply. Like Thomas, Lucas is

⁵ It may not come as a surprise to find many teachers apprehensive of observation and averse to participate in research.

making and taking his time, saying that the shapes are rectangles and, thereby, stating what the instructions had started out with: make different *rectangles* using the string.

- 10 TT:okay you sit down ((*motions to the desk*)) you think of something that they ALL have you put your hand back up. ((*Thomas walks to his seat*)) who can think of something that they ALL HA::VE ((*motions to the cards with her right hand*)) or that they ALL ARE. (1.53) lucas
- 11 L: u::::h (0.81) they are all rectangles,
- 12 (0.36)
- 13 TT:they are all rectangles yes, and why are they all rectangles ((*motions to the cards on the board*)) lucas
- 14 L: u::::h because (0.84)) they (1.69) they kinda width is lo:nger and kinda skinny
- 15 TT:and so if theyre not all the same size around ((holds her hands a distance apart side by side and one on top of the other)) or the length around them we call them oblongs ((points to a rectangle)) right lucas, so theyre all oblongs ((*points to the rectangles*)) theyre all rectangles but theyre all oblongs because none of them are squares (.) what is else is the same about all of them ((*motions to Melissa*)) melissa

Although she is talking a lot, the exchange with Lucas has not taken the lesson any further. But Mrs. Tran is not abandoning. She is giving it another try, this time asking Alycia, and thereby taking additional risks that the sought-after reply will not be forthcoming and that the time invested will have no return.

- 16 M: um that theyre alls that theyre all rectangles
- 17 TT:yes they are all rectangles yes theyre ALL oblongs and theyre ALL rectangles ((*motions to Alycia*)) alycia
- 18 A: um they all have four edges and verticies
- 19 TT:they all have four edges or four sides and they ((*places her right pointer finger along the outside edge of a card*)) all have four verticies or four corners and heres something else (.) they ALL use ((*holds her hands a distance apart*)) the same did we use different loops of string, (.) ((*moves her hands closer then farther apart*)) when we made this, (.) did we did i give you a different did you use a different loop of string ((*moves her right hand in a circle*)) for each rectangle that you made, ((*motions to the board*))

Again, while acknowledging the reply and the veracity of the contents with the actual state of affairs, her continuation shows that she is still after a specific something. She makes a statement that can be heard as a question, “They all use the same . . . did we use different loops of string when we made this?” and she continues, “Did we, did I give you a different, did you use a different loop of string for each rectangle you made?”

Here we also have another clear indication for the fact that thought is not finished and then made available to the audience as if in a memory dump. Instead, there are what we can hear as repeated beginnings: “they all use the same,” “did we use different loops of string when we made this,” “did we,” “did I give you a different,” and “did you use a different loop of string for each rectangle that you made?” We observe here five beginnings, and only the last one completed what we can hear as a question concerning the length of the string and, as a person savvy in mathematics curriculum can hear, implicitly about the perimeter of all the rectangles. Mrs. Tran then receives a first answer, which will

have been the beginning of the end of this final push towards the desired articulation. Many students reply in unison with a “no,” which sets Mrs. Tran up to take another step in producing what will have been a designedly open query: “They were all made with a,” which many children replied to by saying, again in unison, “same” (turn 16). Rather than engaging in the risk of failing to receive the sought-for answer once again, she states what it seems she might have been looking for all along: very *different* rectangles were made using the *same* string, different in terms of their shape (more or less skinny).

20 TT:did you use a different loop of string ((*moves her right hand in a circle*)) for each rectangle that you made, ((*motions to the board*))

21 Ss: no:: ((*some children*))

22 TT:you you all they were all ((*points to the chalkboard with her right hand*)) made with a

23 Ss: same ((*some children*))

24 TT:same piece of string they were all made with the same piece of string ((*places her right hand to her mouth*)) yE:t we made VERY DIFferent rectangles ((*moves her right hand over the rectangles*)) (.) from the sa:me piece of string these some of the rectangles that weve made ((*runs her finger over the first narrow rectangle*)) were long and skinny and then other ones ((*runs her hands under the other rectangles*)) werent so long and skinny and they were more like squares but they werent perfect squares ((*motions to Chris*)) chris ((*time on tape: 00:24:39*))

Mrs. Tran has now almost entirely formulated what appears to have been the observation that she has expected all along: the rectangles have the same perimeter since they where all made using strings of the same length. From turn to turn, narrowing the conversation by means of what look like open-ended questions to a pattern in which students provide straightforward single word answers (“no”, “same”) was part of the risk of engaging in the lesson in that particular way. But there is also richness inherent to all instances of elocution by means of which there is always more in a saying than what is apparently said: We always say more than we can say in so many words (Garfinkel, 1967). There is always a possibility for reinterpretation and elaboration what was really meant, which Mrs. Tran’s last statement exemplifies. From the students’ comments about the being rectangles, she articulates an observation about none of them being squares, which was not part of the lesson’s plan. And in so doing, the teacher does not merely put an end to teaching and learning, but keeps offering something to respond to, even though it is not in the planned form or with the panned content.

The lesson goes on for an additional 8 minutes without some form of statement of potential mathematical interest, such as different areas of rectangles with the same perimeter or the lengths and widths add up to 30, or one length and one width add up to 15. It might have been interesting to find out what the children would have said if asked what the take-home message of the lesson was, what the lesson had shown. This could have made it apparent to the teachers whether they had met their curriculum goal or whether their investment was a loss (write off). Without somehow getting the kids to express themselves, the teachers cannot know how close they have come to the intended goal of this day’s journey. Mrs. Tran and the regular teacher of the class had planned the different tasks to achieve—in a more innovative than normal way—the second-grade curriculum specified in the official curriculum of the province. Despite all their planning, neither one could know in advance whether what they were doing on the basis of the specified plan for the lesson will succeed in the way anticipated. That is, there is an inherent risk that individual parts of the lesson or the lesson as a whole fails in that some or all students will not exhibit what they are intended to exhibit (e.g., consistent with the suggested achievement indicators).

Towards a Risk-Oriented Epistemology

The purpose of this study is to contribute to the building of epistemology generally and to a theory of mathematics education specifically that take into account *risk* as an essential feature of life. Underlying educational uses of institutional curricula and teacher lesson plans with specific learning outcomes are part of a technological rationality (Postman, 1992) of Greco-Roman philosophical origin and tradition (Heidegger, 1982) according to which mathematics students are to learn what has been set out beforehand. The purpose of plans is to eliminate, decrease, or limit risks. In the meeting with the other, in saying the word *Thou*, human beings, as the second and third introductory quotations of this text show, submit themselves to risk and exposure. Human experiences turn out to be in excess of all technological rationality, which has led to the emergence of the concept of risk, a term that has its etymological origins in the post-classical Latin *resicum*, *risicum* in the sense of hazard and danger, itself with possible origins in the Arabic *rizq*, *rēs*q fortune destiny, luck, and chance; these origins are common to many modern languages (see above).

In this study, we suggest that teaching and learning mathematics is characterized by risk as an essential (i.e., irremediable) feature that undermines any technical rationality of curriculum planning. What students learn and what teachers teach is unavoidably uncertain. It remains an empirical matter that can be established with certainty only after the fact—hindsight, as the vernacular saying goes, is 20/20. Life in general and the living mathematics curriculum specifically are not of this nature. In everyday mathematics lessons, students and teachers not only are subjects of activity but also are inherently subject to and subjected to conditions that are also of their making. In the Saying, the one exposes himself/herself to the other (Levinas, 1978) so that, thereby, mathematics teachers and students take and are subject to the risk of failing to achieve the previously stated curriculum (lesson) outcomes (i.e., to non-sense [Merleau-Ponty, 1964]), being hurt. In fact, without risk, there is no teaching and learning; and without teaching and learning, there is no risk.

We use episodes from second-grade geometry lessons to exhibit the inherent unpredictability of the course of the lesson and, therefore, the risk that teachers and students face. The episodes exemplify that teachers and students always act in the face of uncertainty. We show how risk in part derives from exposure to the other, which has implications for mutual and collective responsibility. Teaching, therefore, may also require a *precautionary principle*, much like other phenomena in a risk society. The notion of risk leads us to a rethinking of the subjects of/in mathematics, no longer persons actively constructing themselves, their knowledge, and their situations but *advenants* exposed to the perils of their adventures, which they can only witness but never grasp (Romano, 1998).

One important aspect of a risk-sensitive theory of acting in mathematics concerns the subject. In traditional theories (including constructivism and enactivism), the subject of activity is theorized as an agent who creates (constructs) himself/herself in the autopoietic act of creating the object. However, the subject of experience does not know what s/he will know after going through the experience, and, therefore, also is the *patient* of experience, subject to and subjected to the envining conditions (Dewey, 1934/2008; Vygotskij, 2001). This has led to the articulation of the subject as *advenant*, as someone to whom something advenes. It means that we never are the constructive agents of our experiential selves, as other epistemologies suggest, but we are, passing from self to self, exposed to the risk undoing ourselves in the face to the absolutely other: the event that we can grasp only after it will have been completed (Roth, 2011b). The epistemology we are working towards would have to include the subject as *advenant* and *patient*—capable of being affected, that is, of passibility—in addition to its role of agent and, therefore, agency. As *advenant*, the subject to whom something happens, the person cannot anticipate the results of actions, his/her own and those of others. When Thomas raised his hand,

he could not foresee that the events would put him into a position where he not only did not supply the apparently desired reply but also where his not-knowing was exposed in front of an audience with the possibility of embarrassment. He was put on the spot, not by Mrs. Tran, not by his actions, but by the encompassing situation as a whole that transcends any individual actor present. And similarly, Mrs. Tran could know that she would have to deal with this situation, or the subsequent chain of questions and answers out of which she apparently produced the expected observation—in a text-book case of Jourdain and Topaze effects.

Curriculum planning and the specific learning outcomes that the plans state presuppose the possibility to administratively plan learning. However, if the lesson unfolds in unpredictable and unforeseeable ways, that is, if the learning process takes all sorts of routes, we cannot be certain about what students come to understand, if they come to understand anything at all. This is exemplified in the empirical materials presented above: despite a considerable number of students offering their attempts at replying to Mrs. Tran's question, she ends up articulating it herself. In the course of the students' and teacher's mutual engagement, the signification of the talk is at stake as much as the significance of the array of rectangles visible on the chalkboard (Figure 3). Here, then, the promise of sense that comes with engagement in school mathematics tasks has the non-sense of mathematics as an inherent and necessary companion. Without non-sense, sense could not be distinguished; but the search of sense occurs at the peril of remaining in, and falling back into, non-sense.

In this study we highlight the inherent risks in teaching/learning situations. These risks are not avoided when teachers take complete control over the lesson, because this might fundamentally let teacher talk unfold closer to the way it is planned with the loss of control over what students will take away from the lesson. Acting is exposing oneself to risk—of failure, of getting injured and hurt. Speaking, too, is associated with failure and injury. Speaking, by means of which thinking *realizes* itself (i.e., becomes something of the real world), also is exposition (ex-peau-sition, as one may say or hear in French, outside-of-the-skin-position), and because of the very outside of the position of self, it, too, may get hurt. The detailed analyses exhibit at which point there are risks, such as not producing a reply or even the effect that a statement will have had once we hear the reply. In the very act of listening, too, we face risks, for listening means opening up to something that we do not know in advance, with the possibility that the what (e.g., name) or how of the statement (e.g., ironic, sarcastic) being at the origin of injury, pain, hurt, and suffering. We are subject to conditions that are unpredictable even though we contribute to their making. That is, we are both subjects of the conditions, producing them, as much as being subject to and subjected to them. We see this occurring at all levels of our analysis: individual statement (turn), the episode (Thomas's turn), and the lesson. Both agential and passive aspects are constitutive of risk: Acting is risking. Acting is risking even if we are alone, and it is especially risky when we are with the other, when we address ourselves to the other (Buber, 1970; Nancy, 1993).

Above we note that in (mathematics) education, the notion of risk is generally reserved for students who, because of their class, socio-economic background, race, or gender, experience higher risks of failing than other subgroups of the student population. This study provides empirical examples for the fact that every participant in the educational enterprise is at risk, which in fact is a condition of life (Bakhtin, 1993). It is therefore better to think risk as differential: all students and their teachers are at risk. This means that we can understand how even the most highly achieving students may still not arrive at making sense, being subject to the illusion of sense in the face of apparent non-sense. The very attribution creates another risk: students, parents, and teachers believing that sense has been made when in fact there is nothing but non-sense. It therefore comes as little surprise when even and especially the normally highest achieving students approach their teachers asking questions such as “Am I right so

far?,” “Is this okay?,” or “Am I on track?” As research among older science students has shown, they may do so even and precisely when completely buying into a (radical, social) constructivist paradigm of learning, which means that they may come to think about certain phenomena in ways that will not get them past the next hurdle posed by a (standardized) test (Lucas & Roth, 1996). Precisely because they cannot know what it is they are expected to know after the specified curriculum will have been covered, students cannot know whether what they are doing is somehow getting them there. Only after the fact might they be in a position to state that they have, indeed, arrived where they were intended to arrive, and thus appreciate why they have done what they have done to get to this point of arrival (Roth & Radford, 2011). But in asking their questions, students can, in many cases, be aware of the risks involved in engaging in curriculum tasks: loss of value of investment, possibility of peril to themselves, and being at risk. Not only their understanding of the current mathematics topics or their understanding of mathematics more generally is at risk: their future is at stake (thus, high-stakes testing), making it intelligible that they seek to minimize the risks.

One common practice to deal with the failure of curriculum is to seek fault in students and to blame *them* when the prescribed learning outcomes have not been achieved. Lack of motivation, lack of intelligence (ability), low socio-economic status, family situations (e.g., single parent family), disinterest, and other concepts are used as resources for explaining why an individual lesson or a curriculum as a whole has not led to the planned, prescribed learning outcomes. Less frequently, though still common, teachers are blamed by students, their parents, or educational critics as the reason for low student achievement. In fear of failing to reach the prescribed learning outcomes, many teachers are calling the BC Ministry of Education asking for specific descriptions for how to teach a part of the curriculum (S. Boutonné, personal communication, undated), thereby constituting themselves as technicians following plans for which they are no longer responsible (e.g., Neyland, 2001). This is but one strategy for engaging in risky behavior while securing the possibility for attributing the failure to a source of risk other than one's actions.

This study makes apparent why we require an epistemology that accounts for risk. Our exemplifying empirical materials show that risk exists at many different units of analysis one might choose for looking at mathematics lessons. This is so because the expansion of one's grasp and control over conditions, learning, inherently means risk. Thus, those who want to know from their own experience how a conqueror or discoverer of ideas feels are in need of “*the great health*—one that is such that one cannot only have but also continuous has to acquire and has to acquire, because one constantly exposes it, has to expose it” (Nietzsche, 1954b, p. 258). Nietzsche finds similarities the sciences that predict the future with certainty, the seers who know the future, and god-inspired individuals; in his view, they all are examples of metaphysic endeavors, believe that there is an underlying plan that pre-ordinates everything that can be. The world is not this way, but in continuous renewal through death of the old (ideas) and birth of the new (Bakhtin, 1984b). On this path towards new ideas, we are at risk and we in fact have to expose ourselves to it: there is no other path to the new, which inherently lies beyond the horizon of the unknown, the unseen and therefore unforeseen. In schools, as in academic life more generally, some individuals appear to be willing to take more risks than others, who are not willing to do so. In this study, we suggest that risk is an inherent feature of life generally and of mathematics lessons specifically. We all take and are subject to risk: all of the time. In this study, we repeatedly suggest that acting means risking, and not acting might mean risking even more, to the point that failure becomes certain (the above-noted case of Aurélie). Without participation in mathematics lessons, without engagement in the tasks, and without the associated possibility that the investment of time and emotion will lead to affectively negative outcomes, subjects have no hope in expanding their room to maneuver in the field of study.

Towards an Ethics of Risk in Mathematics Education

To *experience*, as we suggest, means facing *peril*: the risk of failing. The open-endedness of events and experience implies a different kind of ethics than the universal ethics arising from Kantian constructivism (Roth, 2013b). In this section, we outline the issues that an ethics sensitive to risk involves.

In his critique of technocratic mathematics education, Neyland (2001) differentiates technical accountability and ethical responsibility. On the one hand, teachers are externally contrived to behave according to curricular requirements, to follow plans, so that they will be technically cleared from their students' potential failure (or success). In terms of risk, we might say that framing teachers' work in such a way aims at reducing the risk for both teachers and students, asking them to teach and learn according to a mechanistic metaphor where success or failure depends less on individuals' work than on the system as a whole (the tuning of which belongs to the technocrats). Risk is not welcomed in outcome-based curricula: It is, at the best, accepted as inevitable. On the contrary, ethically informed teaching and learning in the spirit of Levinas (1978) hinges on an acknowledgement of risk and uncertainty over an obsessive search for control. Teachers and students are received as inherently ethically responsible for the other prior to and over any form of curricular requirements. In curricular terms, this call for a rethinking of teaching and learning plans in favor of open-ended topics for mathematical investigation. This is a turn that does not exclude students becoming experts of certain mathematical ideas and techniques, but does not organize classroom around that (Neyland, 2010).

Moment-to-moment analysis of teacher-student actions in mathematics lessons from such an ethical perspective then illustrates how, for example, bringing about mathematical concepts or ways of doing have their ethical roots in the dialogical relation of participants in conversation (Maheux & Roth, 2014; Roth, 2013b). When Mrs. Tran insists (through repetition and emphasis through prosody) upon children (and Thomas more specifically) to say something about "*all* the rectangles," she takes up the ethical challenge and offers a topic, a place for dwelling in thinking. More so, the offer is clearly marked as being *for* them, and the question to be investigated *with* her. There is a many-folded risk in doing so, which includes the risk of alienating the other, of missing the ethical possibility of being there with and for the other (prior to curricular "prescribed learning outcome" supposedly demonstrated when students "identify common attributes of triangles, squares, rectangles, and circles from given sets of the same type of 2D shapes" (Figure 1). But this is *also* an exposure, an opening, a possibility for Self and Other to meet in forms of knowing characteristic of mathematics as a culturally and historically developed praxis through which past and distant others also become present (Husserl, 1976).

Mrs. Tran and Thomas responding to one another exhibit being responsible *with* and *for* the other and the condition of being as being-with in even the most banal moment of a mathematics lesson. They exemplify the ethical risk and possibility of doing mathematics with and for an other. And they also permit us to appreciate how mathematics lessons, talk, and statements are risky objects-in-the-making, always open to bifurcations and reinterpretation and so on. To be clear, at the heart of an ethics sensitive to risk—where to experience means facing peril—what matters most is *not* the assessment of teachers actions as best, or good, or appropriate practice. What matters is *not* to decide whether or not Mrs. Tran was "right" to question and answer her students in that way, or if the risk of telling them what she might have hope to hear them say was "worth it." Instead, we conceptualize curriculum as pure mobility, and lessons or utterances as ever-changing moments of events*-in-the making, with ending and outcome yet-to-be-achieved and inherently uncertain and unpredictable (Roth, 2013a). This entails that there cannot be, ever, a final statement or appreciation of human actions, in either objective or subjective terms. Ethics cannot be injected into mathematics lesson plans or preparations, and it cannot be secured. Ethics

is at the tip of every single act, always in the making, dialogical in form and content, always in the coming (Roth, 2013b). For us, it means that growing awareness to the inherent and fundamental presence of risk in teaching and learning is an essential aspect in the advent, in, of, and for mathematics education, of an ethics sensitive to risk as a necessary condition of doing mathematics.

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